DISCUSSION

COMMENTS ON THE ARTICLE "THE OPTIMUM PROFILE OF A RADIATING FIN"*

G. L. Grodzovskii

1. An approximate solution is given in the Solov'ev article for the familiar problem of the optimum profile of a thin radiating fin. The profile of the optimum fin is approximated in the article by a polynomial of third degree whose coefficients were determined by calculation on an M-20 digital computer.

There is no need for such a complex and approximate solution, since we have the exact analytical solution for this problem, as derived by Wilkins [1, 2] and by another method in our paper [3]. Solov'ev provides no literature citations with regard to the Wilkins paper.

2. In the Solov'ev article we find the erroneous contention that in our paper [3] we supposedly introduced: "...as a specified initial quantity the ratio of the temperature at the tip of the fin to the temperature of the fin base. The resulting solution was thus not uniquely defined." On p. 490 we find another erroneous contention to the effect supposedly that "to find the extremal $\varphi(t)$ (the contour of the optimum fin) by analytical means ... is impossible."

In actual fact, in [3] we find the following uniquely defined analytical solution for the problem of the contour of a radiating fin of minimum weight (of minimum cross section F) for a specified initial heat flow Q_0 and a temperature T_0 at the base of the fin.†

The original equations (these naturally coincide with the original equations in the Solov'ev article) are the following:

$$Q = 2\lambda y \frac{dT}{dx}; \tag{1.1}$$

$$\frac{dQ}{dx} = 2\sigma\varepsilon T^4, \tag{1.2}$$

where y is the half-thickness of the fin; the x-axis is directed along the fin; Q is the heat flow along the fin; λ is the thermal conductivity; T is the fin temperature; σ is the Stefan–Boltzmann constant; and ε is the surface emissivity.

An optimum fin contour must ensure a minimum $F = -2\int_0^L y\,dx$. We introduce the dimensionless variables

$$\bar{Q} = \frac{Q}{Q_0}; \quad \bar{T} = \frac{T}{T_0}; \quad \bar{x} = x \Big/ \frac{Q_0}{2\sigma\varepsilon T_0^4};$$

$$\bar{y} = y \Big/ \frac{Q_0^2}{4\lambda\sigma\varepsilon T_0^5}; \quad \bar{F} = F \Big/ \frac{Q_0^3}{8\lambda\sigma^2\varepsilon^2 T_0^9}.$$

*B. A. Solov'ev, Inzhenerno-Fizicheskii Zhurnal, 14, No. 3 (1968).
† Here and below, all notation and numbering of the formulas corresponds to that of [3].

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Then

$$\bar{Q} = \bar{y}\,\frac{d\bar{T}}{d\bar{x}}\,; \tag{1.1'}$$

$$\frac{d\bar{Q}}{d\bar{x}} = \bar{T}^4\,; \tag{1.2'}$$

$$\bar{F} = -2\int_0^{\bar{L}} \bar{y}\,d\bar{x} \tag{1.3'}$$

or

$$\bar{F} = -2\int_1^0 \frac{\bar{Q}\,d\bar{Q}}{\bar{T}^8\,\dfrac{d\bar{T}}{d\bar{Q}}}\,. \tag{1.7'}$$

It follows from the condition of the minimum functional (1.7') that

$$2\bar{Q}\bar{T}\,\frac{d^2\bar{T}}{d\bar{Q}^2} + 16\bar{Q}\left(\frac{d\bar{T}}{d\bar{Q}}\right)^2 - \bar{T}\,\frac{d\bar{T}}{d\bar{Q}} = 0. \tag{1.7}$$

After integration

$$\bar{T} = C_2\,(\bar{Q}^{3/2} + C_1)^{1/9}. \tag{1.8}$$

For the optimum fin contour, proceeding from the variation of the boundary condition with respect to $\bar{T}$, we have $C_1 = 0$, while from the initial condition we have $C_2 = 1$. As a result, the sought solution is written in the form

$$\bar{y} = 6\left(1 + \frac{\bar{x}}{3}\right)^{3.5};\quad \bar{T} = \left(1 + \frac{\bar{x}}{3}\right)^{1/2};\ \bar{Q} = \left(1 + \frac{\bar{x}}{3}\right)^3;\quad \bar{T} = \bar{Q}^{1/6}; \tag{1.9}$$

$$\bar{F}_{\text{opt}} = 8;\quad L_{\text{opt}} = -\frac{3}{2}\,\frac{Q_0}{\sigma\varepsilon\,T_0^4};\quad y_{0\text{opt}} = \frac{3}{2}\,\frac{Q_0^2}{\lambda\sigma\varepsilon\,T_0^5};\ F_{\text{opt}} = \frac{Q_0^3}{\lambda\sigma^2\varepsilon^2\,T_0^9}\,. \tag{1.10}$$

At the end of the optimum fin the temperature $\bar{T}_L$ is uniquely equal to zero.

3. On p. 491 of the Solov'ev article we find an examination of the influence exerted on the weight characteristics by the finite thickness of the fin's edge. The fin contour in this case is specified by rather arbitrary functions. The required reference to the solution of the problem (derived in our paper [3] by means of the Pontryagin method) of the optimum contour for a radiating fin with a specified thickness at the fin's edge (the first erroneous statement by Solov'ev, examined in item 2, apparently refers precisely to this solution of ours). It has been demonstrated rigorously in our paper [3] that the contour of such an optimum fin consists of the segment of constant minimum thickness which adjoins the edge and the segment of the extremal from (1.8). The temperature $\bar{T}_L$ for the edge of the fin is a variable parameter and by no means "specified as an initial quantity"; the optimum value of $\bar{T}_L$ is naturally determined from the condition for minimum $\bar{F}$. In reference [3] we cite an algorithm for the uniquely defined determination of the shape for the optimum contour of a radiating fin with a specified minimum thickness, and an illustrative example is presented in Figs. 2 and 3.

4. At the end of his article, Solov'ev compares the "optimum" fin with one that is triangular. Here again, no reference is made to an exact comparison of optimum, triangular, and rectangular fins, which was given in our paper [3] in Fig. 2.

The comparison by Solov'ev is inexact, since it is based on the approximate profile which, for example, is shorter than the optimum profile of (1.9) by ~25%.

On the whole, the Solov'ev article not only contains no new scientific results, but it misleads the readers by erroneously rejecting the exact solutions, derived many years ago, for the problem under consideration.

LITERATURE CITED

1. Y. Ernest Wilkins, JASS, 27, No. 2 (1960).
2. Y. Ernest Wilkins, J. Soc. Ind. and Appl. Math., 8, No. 4, XII (1960).
3. G. L. Grodzovskii, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Énergetika i Avtomatika, No. 6 (1962); see also: Astronautica Acta, 8, fasc. 4 (1962).

REPLY TO GRODZOVSKII'S COMMENTS

B. A. Solov'ev

In the article entitled "The optimum profile of a radiating fin" [Inzhen.-Fiz. Zh., 14, No. 3 (1968)] I presented an incorrect evaluation of the work of Grodzovskii [Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk Énergetika i Avtomatika, No. 6 (1962)], which properly aroused the indignation of the author of that paper. In connection with this unpleasant incident, I offer my heartfelt apologies to the author of that paper, to the editors, and to the readers of the journal.

The formulation of the problem and its solution by means of a digital computer, as presented in my article through the example of a single nonirradiated radiating fin, can be extended to the optimization of the weight of more complex radiating systems, where no analytical solution is possible. In this connection, I cannot agree with Grodzovskii's comment to the effect that no such formulation of the problem is required. However, the radiating fin optimized with respect to weight by the cited method is virtually in no way inferior to the fin with an exact optimum profile.

The absence of reference to the various papers by Grodzovskii is explained by the fact that it is not the function in a small article, devoted to a restricted problem, to undertake a detailed analysis of all work done in the field.

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